



TITLE:

Nagata spaces and wN -spaces which are preserved by quasi-perfect maps (Set Theoretic and Geometric Topology and Its Applications)

AUTHOR(S):

新田, 眞一; 吉岡, 巖

CITATION:

新田, 眞一 ...[et al]. Nagata spaces and wN -spaces which are preserved by quasi-perfect maps (Set Theoretic and Geometric Topology and Its Applications). 数理解析研究所講究録 2005, 1419: 87-98

ISSUE DATE:

2005-02

URL:

<http://hdl.handle.net/2433/26306>

RIGHT:

Nagata spaces and wN -spaces which are preserved by quasi-perfect maps

四條畷学園短期大学 新田 眞一 (Shin-ichi Nitta)*

Shijonawate-gakuen Junior College

岡山大学・理学部 吉岡 巖 (Iwao Yoshioka)†

Department of Mathematics, Okayama University

1. きっかけとなった問題と否定的解答

Good, Knight and Stares は [3] で、次の命題を示し、それに関連して『 wN -space は quasi-perfect map でその構造が保存されるか?』という問題を提出した。

Proposition 1.1 [3, Proposition 18]. *The closed, finite to one image of a wN -space is a wN -space.*

この問題に対して、[14] で Ying and Good は Lutzer が与えた例;

Example 1.2 [10, Example 4.3]. *A perfect image of a first countable stratifiable space that is not even a q -space.*

が、否定的解答となることを指摘した。それは、Nagata space, wN -space, q -space の定義、そして既に知られている次の事実より明らかである。

Theorem 1.3 [1, Theorem 3.1]. *A space is a Nagata space if and only if it is first countable and stratifiable.*

Lutzer が示した Example 1.2 は、次の事実も示している。

Fact 1.4. *Every quasi-perfect image of any Nagata-space is not Nagata.*

* e-mail address: nitta@jc.shijonawate-gakuen.ac.jp

† e-mail address: yoshioka@math.okayama-u.ac.jp

ところで、次の事実はよく知られている。

Fact 1.5. *The quasi-perfect image of a metrizable space is a metrizable space.*

そこで、ここでは wN -space と metrizable space の間に位置し、quasi-perfect map でその構造が保存される空間を定義し、その空間に関する結果を報告する

2. よく知られている空間の定義

ここでは、space は T_1 -space を、map は continuous で onto map を意味する。space X の subspace A に対し $Cl(A)$ で A の closure を、 N で自然数全体からなる集合を表す。また、ここで特に定義されていない術語などは[2][6]を参照のこと。

Definition 2.1. For a space (X, τ) , a function $g : X \times N \rightarrow \tau$ is called a g -function if $x \in g(x, n)$ and $g(x, n+1) \subseteq g(x, n)$ for each $(x, n) \in X \times N$.

For a subset A of X and $n \in N$, we put $g(A, n) = \bigcup \{ g(x, n) \mid x \in A \}$.

この g -function について、いくつかのよく知られている以下の性質を考える：

- (N) If $g(x, n) \cap g(x_n, n) \neq \emptyset$ for each $n \in N$, then x is a cluster point of the sequence $\langle x_n \rangle$,
- (wN) If $g(x, n) \cap g(x_n, n) \neq \emptyset$ for each $n \in N$, then the sequence $\langle x_n \rangle$ has a cluster point,
- (γ) If $x_n \in g(y_n, n)$ and $y_n \in g(x, n)$ for each $n \in N$, then x is a cluster point of the sequence $\langle x_n \rangle$,
- ($w\gamma$) If $x_n \in g(y_n, n)$ and $y_n \in g(x, n)$ for each $n \in N$, the sequence $\langle x_n \rangle$ has a cluster point,
- (1st) If $x_n \in g(x, n)$ for each $n \in N$, then x is a cluster point of the sequence $\langle x_n \rangle$,
- (q) If $x_n \in g(x, n)$ for each $n \in N$, the sequence $\langle x_n \rangle$ has a cluster point,
- (wM) If $x_n \in g(y_n, n)$, $g(y_n, n) \cap g(z_n, n) \neq \emptyset$ and $z_n \in g(x, n)$ for each $n \in N$, then the sequence $\langle x_n \rangle$ has a cluster point,
- (α) For each $x \in X$, $\bigcap \{ g(x, n) \mid n \in N \} = \{x\}$ holds and, if $y \in g(x, n)$, then $g(y, n) \subseteq g(x, n)$.

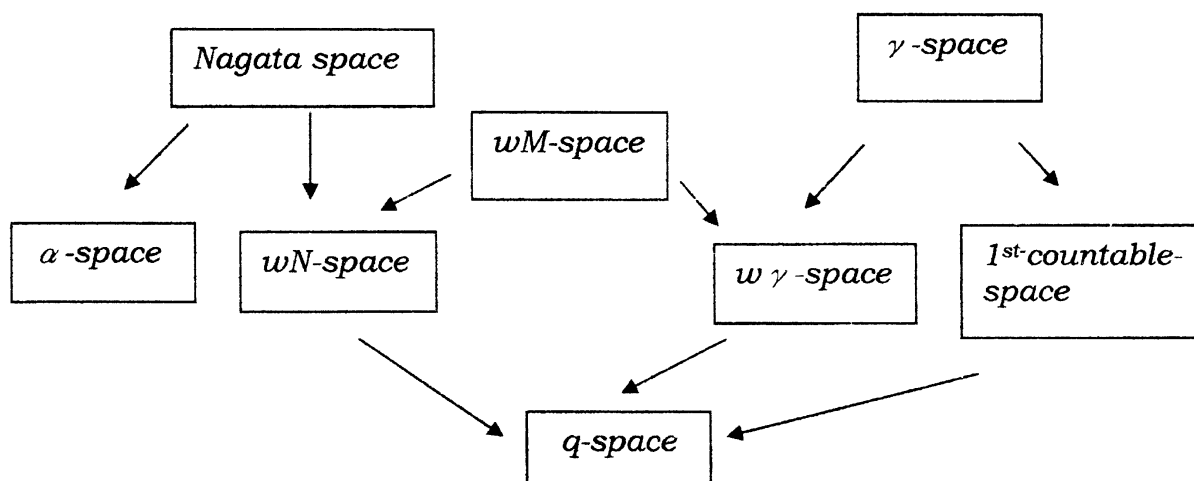
ここでは、 g -function を用いて空間の定義を与える。

Definition 2.2. For a space (X, τ) with a g -function $g : X \times N \rightarrow \tau$,

- (1) X is a *Nagata space* if g satisfies the condition (N),
- (2) X is a *wN -space* if g satisfies the condition (wN),
- (3) X is a *γ -space* if g satisfies the condition (γ),
- (4) X is a *$w\gamma$ -space* if g satisfies the condition ($w\gamma$),
- (5) X is a *1st-countable space* if g satisfies the condition (1st),
- (6) X is a *q -space* if g satisfies the condition (q),
- (7) X is a *wM -space* if g satisfies the condition (wM),
- (8) X is an *α -space* if g satisfies the condition (α).

Nagata space については[1][4][6]、 wN -space、 γ -space、 $w\gamma$ -space、1st-countable space、 q -space についてはそれぞれ[6]、 wM -space については[6][7]、そして α -space については[5]にオリジナルの定義または g -function による特徴づけがある。

これらの空間の関係は次の通りである。



3. Kotake の定理と新しい空間の定義

Nagata space と wN -space の関係について、Kotake は次の定理を示した。

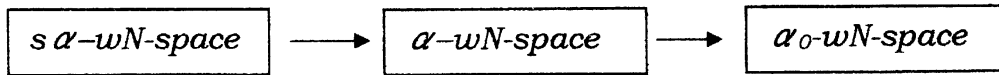
Theorem 3.1[9, Theorem 1.3] *A space is a Nagata space if and only if it is a regular, α and wN -space.*

ここで、 α -space の概念に着目して、 wN -space の構造を含む空間を定義する。

Definition 3.2. For a space (X, τ) ,

- (1) X is called α_0 - wN -space, if there exists a g -function $g: X \times N \rightarrow \tau$ satisfying the conditions (α_0) and (wN) , where
 (α_0) if $y \in g(x, n)$, then $g(y, n) \subseteq g(x, n)$,
- (2) X is called α - wN -space, if there exists a g -function $g: X \times N \rightarrow \tau$ satisfying the conditions (α) and (wN) ,
- (3) X is called $s\alpha$ - wN -space, if there exists a g -function $g: X \times N \rightarrow \tau$ satisfying the conditions $(s\alpha)$ and (wN) , where
 $(s\alpha)$ For for each $x \in X$, $\bigcap \{Cl(g(x, n)) \mid n \in N\} = \{x\}$ holds and,
 if $y \in g(x, n)$, then $g(y, n) \subseteq g(x, n)$.

定義より、これらの空間の関係は次の通りである。



上の定義から α -space に関連して、次の空間が定義される。ここで、strongly α -space は Yoshioka が [15] で定義した。

Definition 3.3. For a space (X, τ) ,

- (1) X is called α_0 -space, if there exists a g -function $g: X \times N \rightarrow \tau$ satisfying the condition (α_0) ,
- (2) X is called *strongly* α -space, if there exists a g -function $g: X \times N \rightarrow \tau$ satisfying the condition $(s\alpha)$.

Remark 3.4. Every strongly α -space is T_2 (hence, $s\alpha$ - wN -space is T_2).

ここで、注意しなければならない事柄は、《 α - wN -space》と《 α , wN -space》は同じ空間を意味しないことである。前者は条件 (α) と (wN) を同時に満たす g -function が存在する空間を、後者は条件 (α) を満たす g -function g と条件 (wN) を満たす g -function h が存在する空間を意味する。

4. $s\alpha$ - wN -space, α - wN -space そして α_0 - wN -space

Definition 3.3 で定義した空間が、よく知られている空間とどのような関係にあるかを調べると、つぎの結果が得られる。

Proposition 4.1. *For a space X , the following statements hold:*

- (1) *If X is a countably compact space, then X is an α_0 - wN -space.*
- (2) *If X is an α_0 - wN -space, then X is a wM -space..*

Proof. (1): For each $(x,n) \in X \times N$, define a g -function $g(x,n)=X$. Then this g -function g satisfies the conditions (α_0) and (wN) .

(2): Let g be g -function satisfying condition (α_0) and (wN) . To show that X is wN , it is sufficient that g -function g satisfies condition $(w\gamma)$, because of [12; Theorem 5.2]. Let $x_n \in g(y_n, n)$ and $y_n \in g(x, n)$ for each $n \in N$, then $g(y_n, n) \subseteq g(x, n)$ by the (α_0) -ness of g -function g . So $x_n \in g(x, n) \cap g(x_n, n)$, and g satisfies the condition (wN) . Thus, $\langle x_n \rangle$ has a cluster point.

上の命題(1)に関連して、countably compact spaces と α_0 - wN -spaces の間に位置する空間について、後で関連する問題として述べる。

また、(2)の逆は成り立つかどうか？ 不明である。

Question 4.2. *Does there exist a wM -space which is not α_0 - wN ?*

なお、 wM -space と α_0 - wN -space の関係については、次が示される。

Proposition 4.3. *Every subparacompact wM -space is an α_0 - wN -space.*

Proof. Let X be a subparacompact wM -space. Since X is wN , X is metacompact by [6; Corollary 3.5]. Since X is wM , there is a sequence $\langle \gamma_n \rangle$ of open covers of X such that $x_n \in \text{st}^2(x, \gamma_n)$ for each $n \in N$, then $\langle x_n \rangle$ has a cluster point (this is the original definition by Ishii [7]). For each $n \in N$, let δ_n be a point-finite open refinement of γ_n . Let $g(x, n) = \bigcap \{U \in \delta_n \mid x \in U\}$ for each $(x, n) \in X \times N$. Then it is easily seen that g -function g satisfies the condition (α) . Now let $g(x, n) \cap g(x_n, n) \neq \emptyset$ for each $n \in N$, then $x_n \in \text{st}^2(x, \delta_n) \subseteq \text{st}^2(x, \gamma_n)$ for each $n \in N$. Thus $\langle x_n \rangle$ has a cluster point, so X is α_0 - wN .

$s\alpha$ - wN -space と α - wN -space については次が成り立つことがわかる。

Theorem 4.4. *For a space X , the following conditions are equivalent:*

- (1) *X is a metrizable space.*
- (2) *X is a $s\alpha$ - wN -space.*

(1) X is a regular α - wN -space.

Proof. (1) \Rightarrow (2): For each $n \in \mathbb{N}$, let $\beta_n = \{B(x; 1/n) \mid x \in X\}$, where $B(x; 1/n)$ is the $1/n$ -neighbourhood of x and let ζ_n be a locally finite closed refinement of β_n . For each $(x, n) \in X \times \mathbb{N}$, define a g -function $g(x, n) = X \setminus \bigcup \{F \in \zeta_n \mid x \notin F\}$. To verify this g -function satisfies condition (N), let $g(x, n) \cap g(x_n, n) \neq \emptyset$ for each $n \in \mathbb{N}$. There exist $y_n \in g(x, n) \cap g(x_n, n)$, $F \in \zeta_n$ and $B \in \beta_n$ such that $x_n, x \in F \subseteq B$. Then $g(x, n) \subseteq \text{st}(x, \beta_n)$ and $x_n \in \text{st}(x, \beta_n)$. It follows that the sequence $\langle x_n \rangle$ clusters at x . So, g satisfies the condition (wN). And it is obvious that this g -function satisfies the condition ($s\alpha$).

(2) \Rightarrow (3): Let g be a g -function with conditions ($s\alpha$) and (wN). To show the regularity of X , let any $x \in X$ and any open set U with $x \in U$. Suppose that for each $n \in \mathbb{N}$, there exist $x_n \in \text{Cl}(g(x, n)) \setminus U$. Then $g(x, n) \cap g(x_n, n) \neq \emptyset$ for each $n \in \mathbb{N}$, so there is a cluster point $p \in X \setminus U$ of the sequence $\langle x_n \rangle$. Now we have for each $n \in \mathbb{N}$, $p \in \text{cl}(\{x_k \mid k \geq n\}) \subseteq \text{Cl}(g(x, n))$, so $p \in \bigcap \{\text{Cl}(g(x, n)) \mid n \in \mathbb{N}\} = \{x\}$. Hence $p = x$, this is a contradiction.

(3) \Rightarrow (1): Since X is regular α and wN -space, X is Nagata by Theorem 3.1. And X is wM by Proposition 4.1(2), it follows that X is $w\gamma$. By [6; Theorem 4.7], X is metrizable.

$s\alpha$ - wN -space (=metrizable space), α - wN -space として α_0 - wN -space の概念は互いに異なるものであることが、次の例によって示される。

Example 4.5. There exists an α - wN -space which is not $s\alpha$ - wN .

Proof. Let X be the space \mathbb{N} with the cofinite topology $\{U \subseteq \mathbb{N} \mid |\mathbb{N} \setminus U| \text{ is finite}\} \cup \emptyset$. It is well known that X is compact T_1 but not T_2 . Since X is not T_2 , X is not $s\alpha$ - wN . To show that X is a α - wN -space, let $g(x, n) = \{x\} \cup \{k \geq n\}$ for each $(x, n) \in X \times \mathbb{N}$. Then g is a g -function satisfying the condition (α) and by the compactness of X , g satisfies the condition (wN) (see [15; Example 4.10]).

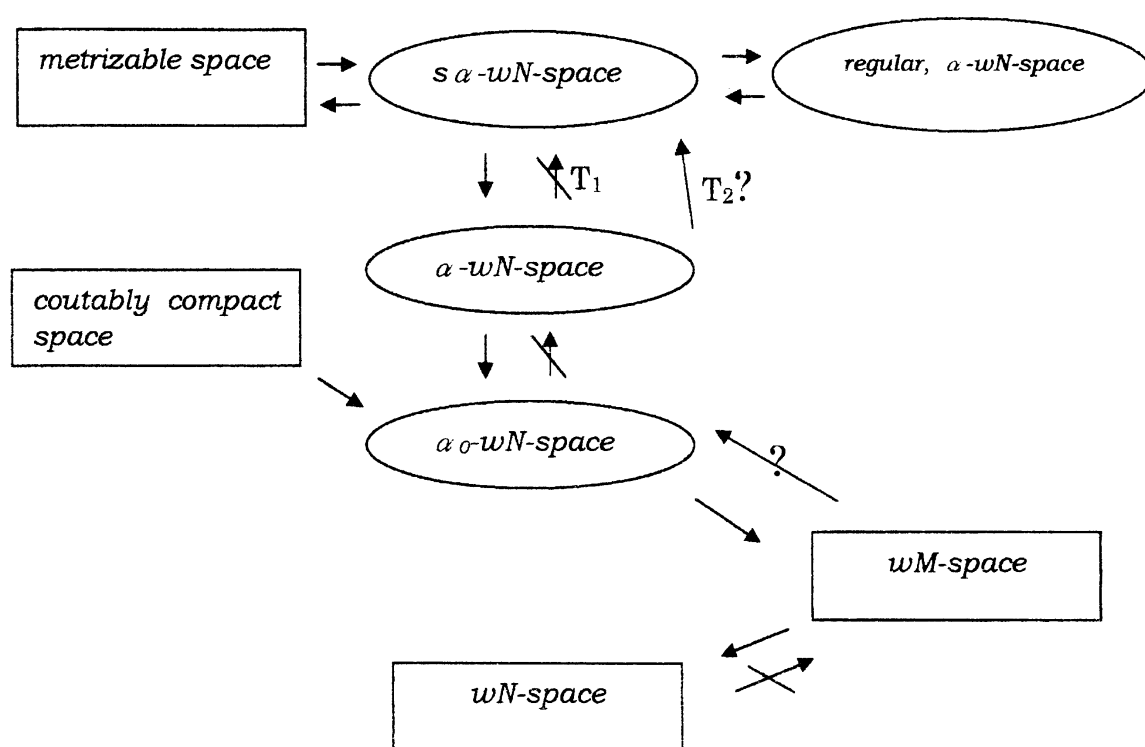
Example 4.5 の空間は T_2 -space でない。Theorem 4.4 の結果より次の質問が考えられる。

Question 4.6. Is every T_2 α - wN -space, metrizable?

Example 4.7. There exists an α_0 - wN -space which is not α - wN .

Proof. Let X be the space of all countable ordinal numbers with order topology. Since X is countably compact, X is α_0 - wN by Proposition 4.1(1). Suppose that X is an α - wN -space. Then X is metrizable by Theorem 4.4. This is a contradiction.

このセクションの内容は次のようになる。



なお、 wN -space であつて、 wM でない空間の例は Remark 5.4 を参照のこと。

5. α - wN -space および α_0 - wN -space の quasi-perfect image

Theorem 4.4 より $s\alpha$ - wN -space は metrizable であるから $s\alpha$ - wN -space の quasi-perfect image は $s\alpha$ - wN -space (=metrizable space) である。

α_0 - wN -space および α - wN -space の quasi-perfect image については次の定理が成り立つ。

Theorem 5.1. *The following statements hold:*

- (1) *The quasi-perfect image of an α_0 - wN -space is α_0 - wN .*
- (2) *The quasi-perfect image of an α - wN -space is α - wN .*

Proof. (1): Let $f: X \rightarrow Y$ be a quasi-perfect map, and X an α - ω N-space. The space X has a g -function g satisfying the conditions (α_0) and (ωN) . Let $h(y, n) = Y \setminus f(X \setminus g(f^{-1}(y), n))$ for each $(y, n) \in Y \times N$, then Y has a g -function h by the closedness of f . We will show that h satisfies the condition (α_0) and (ωN) . Let $z \in h(y, n)$, then for each $u \in f^{-1}(z)$ there exists $x_u \in f^{-1}(y)$ such that $u \in g(x_u, n)$. Since g satisfies the condition (α_0) , $g(u, n) \subseteq g(x_u, n) \subseteq g(f^{-1}(y), n)$. Then $g(f^{-1}(z), n) \subseteq g(f^{-1}(y), n)$. It follows that $h(z, n) \subseteq h(y, n)$ holds. Next to verify that h satisfies the condition (ωN) , let $z_n \in h(y, n) \cap h(y_n, n)$ for each $n \in N$. Let $n \in N$, there exist u_n and w_n such that $u_n \in f^{-1}(y)$ and $w_n \in f^{-1}(z_n) \cap g(u_n, n)$, and since $z_n \in h(y_n, n)$, there is an $x_n \in f^{-1}(y_n)$ such that $w_n \in g(x_n, n)$. The sequence $\langle u_n \rangle$ has a cluster point p in $f^{-1}(y)$ by the countable compactness of $f^{-1}(y)$. For each $k \in N$, there is a $u_{n(k)} \in g(p, k)$ with $n(k) < n(k+1)$. Since g satisfies the condition (α_0) , $g(u_{n(k)}, n(k)) \subseteq g(p, n(k)) \subseteq g(p, k)$ hold. Then $w_{n(k)} \in g(p, k) \cap g(x_{n(k)}, k)$ for each $k \in N$. Since g satisfies the condition (ωN) , the sequence $\langle x_{n(k)} \rangle$ clusters, so $\langle x_n \rangle$ has a cluster point. Thus $\langle f(x_n) \rangle$ has a cluster point, that is, $\langle y_n \rangle$ has a cluster point.

(2): Let $f: X \rightarrow Y$ be a quasi-perfect map, and X an α - ω N-space. The space X has a g -function g satisfying the conditions (α) and (ωN) . Let $h(x, n) = Y \setminus f(X \setminus g(f^{-1}(x), n))$ for each $n \in N$, then Y has a g -function h by the closedness of f . We will show that h satisfies the conditions (α) and (ωN) . From the proof of (1), we only show that $\bigcap \{h_n(y) \mid n \in N\} = \{y\}$ for each $y \in Y$. Suppose that there is a $y \in Y$ such that $h_n(y) \neq \{y\}$. Then there is an $x \in \bigcap \{g(f^{-1}(y), n) \mid n \in N\} \setminus f^{-1}(y)$. For each $n \in N$, there exists $x_n \in f^{-1}(y)$ such that $x \in g(x_n, n)$. The sequence $\langle x_n \rangle$ has a cluster point p in $f^{-1}(y)$ by the countable compactness of $f^{-1}(y)$. For each $k \in N$, there is an $n(k) \in N$ such that $x_{n(k)} \in g(y, k)$ with $n(k) < n(k+1)$. Then $x \in g(x_{n(k)}, n(k)) \subseteq g(x_{n(k)}, k) \subseteq g(p, k)$ hold. Thus we have $x \in \bigcap \{g(p, n) \mid n \in N\} = \{p\}$, so $x = p$. This is a contradiction.

Remark 5.2. The fact that the quasi-perfect image of an α -spaces is also α can be shown in the same manner as the proof of Theorem 5.1.

Remark 5.3. In [8], Ishi showed that the quasi-perfect image of a ωM -space is also a ωM -space.

Remark 5.4. As stated in section 1, Lutzer showed in [10, Example 4.3] there exists a perfect map $f: X \rightarrow Y$ where X is a Nagata space and Y is not q -space. We can find this space X is not ωM by Proposition 4.3 and Theorem 5.1(1). Thus, this space X is a ωN space which is not ωM .

ところで, metrizable が closed map で保存されないことはよく知られている。

Example 5.5 (see [11, Example 10.1]). A closed image of a metrizable space is not a q -space.

この例は, $s\alpha$ - ωN , α - ωN そして α_0 - ωN のそれぞれの性質は closed map で保存されないことをも示している。

6. Nagata space の場合

これまでは ωN -spaces に関して述べてきた。同様な事柄を Nagata-space について調べてみる。

Definition 6.1. For a space (X, τ) ,

X is called an α_0 -Nagata space, if there exists a g -function $g: X \times N \rightarrow \tau$ satisfying the conditions (α_0) and (N) .

Theorem 6.2. For a space X , the following conditions are equivalent:

- (1) X is a metrizable space.
- (2) X is an α_0 -Nagata space.

Proof. (1) \Rightarrow (2): In the proof of Theorem 4.4(1) \Rightarrow (2), we have shown this implication.

(2) \Rightarrow (1): Let g be a g -function satisfying the condition (α_0) and (N) . We will show that g satisfies the condition (γ) . Let $x_n \in g(y_n, n)$ and $y_n \in g(x, n)$ for each $n \in N$, then $x_n \in g(y_n, n) \subseteq g(x, n)$, because g satisfies (α_0) . Since $x_n \in g(x, n) \cap g(x_n, n)$ and g satisfies the condition (N) , the sequence $\langle x_n \rangle$ clusters at x . Thus, X is γ . By [6; Theorem 4.7], X is metrizable.

Definition 3.2 (2), (3) と同様に、 α -Nagata space と $s\alpha$ -Nagata space を定義することは可能である。しかし、Theorem 4.4 の証明より、 α -Nagata space および $s\alpha$ -Nagata space は metrizable であることがわかる。

Nagata space と ωN -space の関係について述べられていた Theorem 3.1 と同様な定理が成り立つ。

Theorem 6.3. A space is a Nagata space if and only if it is a strong α , ωN -space.

Proof. Let X be a Nagata space. We will show that X is strong α . Since every Nagata space is semi-stratifiable and paracompact T_2 , X has a G_δ -diagonal sequence $\langle \delta_n \rangle$. And for each $n \in \mathbb{N}$, there is a locally finite closed refinement ζ_n of δ_n . For each $(x, n) \in X \times \mathbb{N}$, let $g(x, n) = X \setminus \bigcup \{F \in \zeta_n \mid x \notin F\}$. Then it is easily verified that if $y \in g(x, n)$, then $g(y, n) \subseteq g(x, n)$ holds. We will show that $x \in \bigcap \{\text{Cl}(g(x, n)) \mid n \in \mathbb{N}\} = \{x\}$ for each $x \in X$. For any $y \in X$ with $y \neq x$, there is an $m \in \mathbb{N}$ such that $y \notin \text{st}(x, \zeta_m)$. Then we have $g(x, m) \cap g(y, m) = \emptyset$. Suppose that there is a $z \in g(x, m) \cap g(y, m)$, then $z \in F \in \zeta_m$. Since ζ_m is a refinement of δ_m , we have $x, y \in F \subseteq G$ for some $G \in \delta_m$. Then we have that $y \in \text{st}(x, \delta_m)$, this is a contradiction. Thus, X is a strongly α , ωN -space.

Conversely, Let X be a strongly α , ωN -space. Let h be a g -function satisfying condition $(s\alpha)$, and k g -function satisfying condition (wN) . Let $g(x, n) = h(x, n) \cap k(x, n)$ for each $(x, n) \in X \times \mathbb{N}$, then g is a g -function. We will verify that X is regular. Suppose that U be an open neighbourhood of x and $x_n \in \text{Cl}(g(x, n)) \setminus U$ for each $n \in \mathbb{N}$. Since $k(x, n) \cap k(x_n, n) \neq \emptyset$, the sequence $\langle x_n \rangle$ has a cluster point $p \notin X \setminus U$. And $p \in \text{cl}\{\{x_k \mid k \geq n\} \subseteq \text{Cl}(g(x, n))$ for each $n \in \mathbb{N}$. On the other hand, $\bigcap \{\text{Cl}(h(x, n)) \mid n \in \mathbb{N}\} = \{x\}$, since h satisfies the condition $(s\alpha)$. Then, $p \in \bigcap \{\text{Cl}(g(x, n)) \mid n \in \mathbb{N}\} \subseteq \bigcap \{\text{Cl}(h(x, n)) \mid n \in \mathbb{N}\} = \{x\}$, so we have $x = p$. This is a contradiction. Thus, X is a regular α , ωN -space. By Theorem 3.1, X is Nagata.

7. 関連する問題

Proposition 4.1(1) で countably compact space は α_0 - ωN -space であることを述べた。countably compact の一般化された空間としてよく知られているものとして Morita が定義した M -space がある。さらに、 M -space の一般化として、Siwiec and Nagata は [13] において $M^\#$ -space を定義した。

Definition 7.1. A space X is called an $M^\#$ -space if it has a sequence $\{\zeta_n\}$ of closure-preserving closed covers of X such that whenever $x_n \in \text{st}(x, \zeta_n)$ for each $n \in \mathbb{N}$, then the sequence $\langle x_n \rangle$ has a cluster point.

$M^\#$ -space に対して次の命題が成り立つ。

Proposition 7.2. Every $M^\#$ -space is an α_0 - ωN -space.

Proof. Let $\langle \zeta_n \rangle$ be a sequence of closure-preserving closed covers of X such that whenever $x_n \in \text{st}(x, \zeta_n)$ for each $n \in \mathbb{N}$, then the sequence $\langle x_n \rangle$ has a cluster point, where we may assume that ζ_{n+1} is a refinement of ζ_n . Let $g(x, n) = X \setminus \bigcup \{F \in \zeta_n \mid x \notin F\}$ for each $n \in \mathbb{N}$, then

the g -function g satisfies condition (α_0) . And let $g(x, n) \cap g(x_n, n) \neq \emptyset$ for each $n \in \mathbb{N}$, then $x_n \in \text{st}(x, \zeta_n)$ for each $n \in \mathbb{N}$. So the sequence $\langle x_n \rangle$ has a cluster point. Thus X is α_0 - wN .

上の命題について、逆が成り立つかどうか不明である。

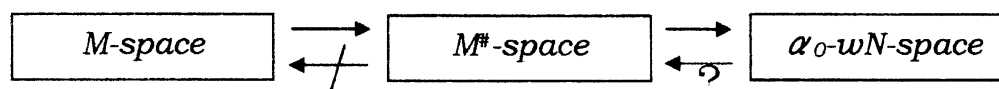
Question 7.3. Does there exist an α_0 - wN -space which is not $M^\#$?

なお、 α_0 - wN -space と $M^\#$ -space の関係については、次が示される。

Proposition 7.4. Every paracompact T_2 , wM -space is an $M^\#$ -space.

Proof. Let X be a paracompact T_2 , wM -space. Let $\{\gamma_n\}$ be a sequence of open covers of X such that $x_n \in \text{st}^2(x, \gamma_n)$ for each $n \in \mathbb{N}$, then $\langle x_n \rangle$ has a cluster point. For each $n \in \mathbb{N}$, there is a locally finite closed refinement ζ_n of γ_n . Then the sequence $\{\zeta_n\}$ is the one of closure-preserving closed covers of X such that whenever $x_n \in \text{st}(x, \zeta_n)$ for each $n \in \mathbb{N}$, then the sequence $\langle x_n \rangle$ has a cluster point. Thus X is an $M^\#$ -space.

M -space, $M^\#$ -space そして α_0 - wN -space の関係は次の通りである。



$M^\#$ -space であって M -space でない例は[12]で示されている。

最後に、 wN -space および Nagata space の距離化について、Hodel が示した定理に着目する。

Theorem 7.5. The following statements hold:

- (1)[6;Theorem 4.3] Every T_2 γ , wN -space is metrizable.
- (2)[6;Theorem 4.7] Every $w\gamma$, Nagata space is metrizable.

この定理より、次の2つの問題は Question 4.6 と同値な問題となる。

Question 7.6. Is every T_2 α - wN -space, Nagata ?

Question 7.7. Is every T_2 α - wN -space, γ ?

References

- [1] Ceder J.G, Some generalizations of metric spaces, Pacific J.Math., 11 (1961), 105-125,
- [2] Engelking R, General Topology, Polish Sci.Publ., Warsaw,1977.
- [3] Good C, Knight R and Stares I, Monotone countable paracompactness, Topology and its Appl.,101 (2000), 281-298.
- [4] Heath R.W, On open mappings and certain spaces satisfying the first countability axioms, Fund.Math.,57 (1965), 91-96.
- [5] Hodel R.E, Moore spaces and $w\Delta$ -spaces, Pacific J.Math.38 (1971), 641-652.
- [6] Hodel R.E., Spaces defined by sequences of open covers which gurantee that certain sequences have cluster points, Duke Math.,39 (1972), 253-263.
- [7] Ishii T, wM -spaces I ,II , Proc.Japan Acad.,46 (1970), 5-10,11-15.
- [8] Ishii T, wM -spaces and closed maps, Proc.Japan Acad.,46 (1970), 16- 21.
- [9] Kotake Y, On Nagata spaces and wN -spces, Science Reports of the Tokyo Kyoiku Daigaku, Sec.A,12 (1973), 46-48.
- [10] Lutzer J, Semi-metrizable and stratifiable spaces, General Top.and its Appl.,1 (1971), 43-48.
- [11] Michael E, A quintuple quotient quest, General Top. And its Appl., 2 (1972), 91-138.
- [12] Morita K, Products of normal spaces with metric spaces,Math.Ann., 154 (1964), 365-382
- [13] Siwiec F and Nagata J, A note on nets and metrization, Proc.Japan Acad.,44 (1968), 623-627.
- [14] Ying G and Good C, A note on monotone countable paracompactness, Comment.Math.Univ.Carolinae,42 (2001), 771-778.
- [15] Yoshioka I, Closed images of spaces having g-functions, Topology and its Appl., to appear.